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10MMD/MDE/MCM/MEA/MAR11

First Semester M.Tech. Degree Examination, January 2011

Applied Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Discuss:
i) Absolute error
ii) Truncation error
iii) True percentage relative error
iv) Conservation laws of engineering. **(10 Marks)**

b. Derive the analytical solution as $v(t) = \frac{mg}{c}\left[1 - e^{-(c/m)t}\right]$ for the differential equation $\frac{dv}{dt} = g - \left(\frac{c}{m}\right)v$ where m is the mass of the falling body (parachutist), C is the drag coefficient, g – gravitational force, v is the velocity and t is the time. Also discuss the terminal velocity of the parachutist. **(10 Marks)**

2 a. Explain Regula-Falsi method to find the root of the equation. Apply Newton-Raphson method to find the root of the equation $\cos x - xe^x = 0$ near x = 1 (carry out 5 approximations with 4 decimals). **(10 Marks)**

b. Explain Newtons method for finding the multiple roots of the equation f(x) = 0. Find the double root of the equation $x^3 - x^2 - x + 1 = 0$ at x = 0.9, x = 0.8. **(10 Marks)**

3 a. Give the necessary steps to find the roots of polynomial by Muller's method. Find the root of the equation $x^3 - 3x - 5 = 0$ which lies in [2, 3]. **(10 Marks)**

b. Perform two iterations of the Baristow's method to extract a quadratic factor $x^2 + px + q$ from the polynomial $p_3(x) = x^3 + x^2 - x + 2 = 0$. Use p_0 = –0.9 and q_0 = 0.9 as initial approximations. **(10 Marks)**

4 a. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1.2, 2.0, 2.2 using numerical differentiation given that :

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.025

(10 Marks)

b. Give the steps to integrate $\int_a^b f(x)dx$ using Romberg integration. Use it to find the approximate value of $\int_0^1 \frac{dx}{1+x}$. Take h = 0.5, 0.125 and 0.25. **(10 Marks)**

5 a. Discuss Cholesky method to solve the system of linear equations. Solve $2x + 3y + z = 9$, $x + 2y + 3z = 6$ and $3x + y + 2z = 8$ by Cholesky method. **(10 Marks)**

b. Determine the inverse of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$ using partition method. Hence find the solution of the system of equations $x_1 + x_2 + x_3 = 1$, $4x_1 + 3x_2 - x_3 = 6$ and $3x_1 + 5x_2 + 3x_3 = 4$. **(10 Marks)**

6 a. Discuss : i) Bounds for eigen values

ii) Steps involved in Jacobi iteration method to find eigen values. **(10 Marks)**

b. Find all the eigen values of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 2 \end{bmatrix}$ using Rutishausen method. (Take five stages). **(10 Marks)**

7 a. i) State the properties of linear transformation.

ii) The columns of $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ are $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Suppose T is a linear transformation from R^2 to R^3 such that $T(e_1) = \begin{bmatrix} 5 \\ -7 \\ 2 \end{bmatrix}$ and $T(e_2) = \begin{bmatrix} -3 \\ 8 \\ 0 \end{bmatrix}$, find the image of an arbitrary x in R^2. **(08 Marks)**

b. If $T : R^n \rightarrow R^m$ be a linear transformation, prove that :

i) T is one-to-one if and only if the equation T(X) = 0 has trivial solution.

ii) T maps R^n onto R^m if and only if the columns of A span R^m.

iii) T is one to one if and only if the columns of A are linearly independent. **(12 Marks)**

8 a. i) Give a geometrical interpretation of the orthogonal projection.

ii) If $u_1 = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$, $u_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$, $y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\{u_1 \; u_2\}$ is an orthogonal basis for $w = \text{span}\{u_1, u_2\}$, write y as the sum of a vector in w and a vector orthogonal to w. **(10 Marks)**

b. Discuss : i) Gram-Schmidt process

ii) Least square lines

iii) The general linear model. **(10 Marks)**

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